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Canonical Path-Integral Quantization of Yang–Mills Field Theory with Arbitrary External Sources

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Some modification of source terms is proposed for gauge field theories. In the $SU(2)$ Yang–Mills theory with arbitrary external sources a canonical quantization procedure leads to a Lorentz-invariant S-matrix only when Fermi statistics is imposed on ghost fields. The usual source terms lead to a result that breaks Lorentz invariance and is singular when external charges J_0^a vanish. The cases of the Abelian scalar electrodynamics and the $SU(2)$ Yang–Mills field with external currents ($J_0^a = 0$, $J_i^a \neq 0$) are also discussed.

1. INTRODUCTION

In the present paper we carry out the canonical path-integral quantization of the $SU(2)$ Yang–Mills field theory in the presence of external sources. We stress that some modification has to be made in order to arrive at a *plausible* formula for the S-matrix. The usual source terms (Schwinger, 1951), when quantized canonically, lead to a sharp discrepancy between the case of vanishing external charge (Cabo and Shabad, 1986) and the case of nonzero external charge (Kiskis, 1980).

In their recent paper Cabo and Shabad (1986) quantize the Yang–Mills field theory with external sources, but they carry out the precise procedure only for the case of zero external charges $J_0^a = 0$. They use some heuristic arguments to generalize their partial result, and some doubts remain. It seems to us that the canonical procedure should be carried out also in the generic case $J_\mu^a \neq 0$. Then one could decide which conclusion, that of Cabo and Shabad (1986) or that of Kiskis (1980), is correct.

We propose to start with a new, modified Lagrangian, which, by the way, is the effective Lagrangian of Cabo and Shabad. Then we analyze the

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canonical structure of this new theory; in particular we find all constraints. Further, we define the generating functional (S-matrix) as the path integral over *the almost unstrained phase space.* At last, after some calculational tricks, we arrive at the desired result—*the relativistic formula for the S-matrix.*

One could object that this is just a tautology—we have obtained the result imposed earlier. But it turns out that our final formula substantially depends on the statistics of the subsidiary scalar fields. If we had treated them all as bosonic variables, then the final formula would lose both Lorentz invariance and BRS supersymmetry (Becchi *et al.*, 1976). This is a completely new phenomenon, because the canonical quantization procedure in the usual covariant gauge leads to a Lorentz-invariant S-matrix formula (Kugo and Oijima, 1979), no matter what statistics ghost fields obey.

All this leads us to the conclusion that our modification is *plausible* and that it is worthwhile to study this new theory. Maybe we are still far from physical reality, but the theory possesses some very interesting features which should be analyzed. We argue that the previous attempts to implement naive non-Abelian Gauss laws as constraints on physical states (Kiskis, 1980; Goldstone and Jackiw, 1978; Das *et al.*, 1979) were all doomed from the start—one should start with a different modified Lagrangian. Further physical consequences of our modification are the subject of our current research.

2. CANONICAL QUANTIZATION OF THE $SU(2)$ YANG–MILLS FIELD THEORY WITH MODIFIED SOURCE TERMS

The naive source terms (Schwinger, 1951) seem to be unfounded in the case of gauge fields; however, their presence breaks gauge symmetry and the degeneracy of the theory seems to be removed (Białynicki-Birula and Przeszowski, 1987). This is why we would not like to follow the gauge-invariant source term prescription (Fradkin and Tseytlin, 1984; Vilkogheky, 1984). Instead, we propose to introduce some subsidiary scalar fields, which play the roles of ghosts and Lagrange multipliers. The modified Lagrangian should be a Lorentz-invariant quantity and should obey the same gauge transformation laws as the naive one does (Białynicki-Birula and Przeszowski, 1987).

We propose to start with following Lagrangian density:

$$L_{\text{mod}} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + A^a_\mu J^{a\mu} + Q^a D^{ab}_\mu J^{b\mu} + \varepsilon^a b^c \bar{c}^a J^{b\mu} D^{cd\mu} c^d \tag{1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu$$

$$D^{ab}_\mu = \partial_\mu \delta^{ab} + g\varepsilon^{acb} A^c_\mu$$

and to treat all fields as the independent variables. The above formula is the effective Lagrangian of Cabo and Shabad (1986). Here we are not going to repeat their heuristic arguments in order to prove its correctness—we just take it as a possible starting point.

Before we reach the heart of our problem, we first study the gauge symmetry properties of our Lagrangian (1). One can impose a gauge transformation by means of a unitary $SU(2)$ matrix U:

$$A^a_\mu \lambda^a \to U^{-1}\left(\frac{1}{g}\partial_\mu + A^a_\mu \lambda^a\right)U \tag{2a}$$

$$J^a_\mu \lambda^a \to U^{-1} J^a_\mu \lambda^a U \tag{2b}$$

$$Q^a \lambda^a \to U^{-1} Q^a \lambda^a U \tag{2c}$$

$$\bar{c}^a \lambda^a \to U^{-1} \bar{c}^a \lambda^a U \tag{2d}$$

$$c^a \lambda^a \to U^{-1} c^a \lambda^a U \tag{2e}$$

where λ^a, the $SU(2)$ group generators, are normalized by the condition

$$\mathrm{Tr}\{\lambda^a \lambda^b\} = \delta^{ab} \tag{2f}$$

Now one can easily find that under such transformation L_{mod} changes only by the field-independent factor:

$$L_{\mathrm{mod}} \to L_{\mathrm{mod}} - \frac{1}{g}\mathrm{Tr}[(\partial_\mu U^{-1}) J^{a\mu} \lambda^a U] \tag{3}$$

This remarkable property allows us to simplify considerably our quantization procedure. One can choose $U = U_0$, where the matrix U_0 parametrizes external sources J^a_μ (Białynicki-Birula and Przeszowski, 1987):

$$J^a_\mu = U_0^{-i} j^a_\mu \lambda^a U_0 \tag{4}$$

Here j_μ is no longer an arbitrary quantity, but some conditions are to be imposed on it—those conditions implicitly define U_0. For our canonical quantization program it is prudent to choose the following condition:

$$j^a_0(x) = \delta^{a3} \rho(x) \tag{5}$$

Now we can change the names of the subsidiary fields, c^3 and c^{-3} into η and $\bar{\eta}$, respectively, and use superscripts $\bar{a}, \bar{b}, \ldots$ for the color indices 1, 2. All this allows us to write the modified Lagrangian density in the following form:

$$\begin{aligned}\bar{L}_{mod} = &-\tfrac{1}{4} D^a_{\mu\nu} F^{a\mu\nu} + A^3_0 \rho - A^a_i j^a_i + Q^a[g\rho\varepsilon^{\bar{a}\bar{b}} A^{\bar{b}}_0 - S^{\bar{a}\bar{b}}_i j^{\bar{b}}_i] \\ &+ \rho[\varepsilon^{\bar{a}\bar{b}} \bar{c}^{\bar{b}} \partial_0 c^{\bar{a}} + g\bar{c}^{\bar{a}} A^{\bar{a}}_0 \eta - g A^3_0 \bar{c}^{\bar{a}} c^{\bar{a}}] + Q^3[\dot{\rho} - D^{3b}_i j^b_i] \\ &- \bar{c}^{\bar{a}} m^{\bar{a}\bar{b}} c^{\bar{b}} - \bar{c}^{\bar{a}} m^{\bar{a}3} \eta - \bar{\eta} m^{3\bar{a}} c^{\bar{a}} - \bar{\eta} m^{33} \eta\end{aligned} \tag{6}$$

where we have introduced new symbols:

$$\varepsilon^{\bar{a}\bar{b}} = \varepsilon^{3\bar{a}\bar{b}} \tag{6a}$$

$$m^{ab} = \varepsilon^{acd} j_i^c D_i^{db} \tag{6b}$$

Because $\bar{L}_{\rm mod}$ differs from $L_{\rm mod}$ only by the field-independent factor, we can begin the canonical quantization procedure with $\bar{L}_{\rm mod}$. First we look for the canonical conjugate momenta:

$$E_i^a(x) := \delta S/\delta \dot{A}_i^a(x) = F_{0i}^a(x) \tag{7a}$$

$$E_0^a(x) := \delta S/\delta \dot{A}_0^a(x) \equiv 0 \tag{7b}$$

$$\pi^{\bar{a}}(x) := \delta S/\delta \dot{c}^{\bar{a}}(x) = \mp \varepsilon^{\bar{a}\bar{b}} \rho(x) \bar{c}^{\bar{b}}(x) \tag{7c}$$

$$q^a(x) := \delta S/\delta \dot{Q}^a(x) \equiv 0 \tag{7d}$$

$$\pi^3(x) := \delta S/\delta \dot{\eta}(x) \equiv 0 \tag{7e}$$

$$\bar{\pi}^a(x) := \delta S/\delta \dot{\bar{\eta}}(x) \equiv 0 \tag{7f}$$

where the action S is defined as

$$S := \int d^4x\, \bar{L}_{\rm mod}(x)$$

Dots stand for the time derivatives and in equation (7c) the minus (plus) sign corresponds to the case of fermion (boson) fields c and $\bar{c}$. In the first case c and $\bar{c}$ are to be represented by the Grassmann variables and a minus sign comes straightforwardly from the differentiation rules for such variables.

Equations (7b) and (7d)–(7f) describe the primary constraints of our modified Lagrangian. This means that the fields A_0^a, Q^a, η, and $\bar{\eta}$ are nondynamical and they should be eliminated from the truly unconstrained dynamics. However, we will use the Dirac prescription (Dirac, 1950, 1959, 1964), which systematically removes the redundant degrees of freedom. Thus, we build the Hamiltonian density using the usual formula and introduce the Lagrange multipliers (u^a, v^a, λ, $\bar{\lambda}$) for the primary constraints:

$$\begin{aligned}
\mathcal{H} &:= A_i^a E_i^a + \dot{c}^{\bar{a}} \pi^{\bar{a}} - \bar{L} + v^a E_i^a + u^a q^a + \bar{\lambda} \bar{\pi}^3 + \lambda \pi^3 \\
&= \tfrac{1}{2}(E_i^a)^2 + \tfrac{1}{2}(B_i^a)^2 + A_i^a j_i^a - A_0^a[D_i^{ab} E_i^a + \rho\delta^{a3}] \\
&\quad - Q^{\bar{a}}[\varepsilon^{\bar{a}\bar{b}} \rho A_0^{\bar{b}} - D_i^{\bar{a}b} j_i^a] + Q^3[\dot{\rho} - D_i^{3b} j b_i^b] \\
&\quad \mp g\pi^{\bar{a}} \varepsilon^{\bar{a}\bar{b}}[A_0^3 c^{\bar{b}} - A_0^{\bar{b}} \eta] \mp (\pi^{\bar{a}}/\rho) \varepsilon^{\bar{a}\bar{b}}[m^{\bar{n}b\bar{c}} v^{\bar{c}} + m^{\bar{b}3} \eta] \\
&\quad + \bar{\eta}[m^{3\bar{a}} c^{\bar{a}} + m^{33}\eta] + v^a E_i^a + \bar{\lambda}\pi^3 + \lambda\pi^3
\end{aligned} \tag{8}$$

where $B_i^a = \frac{1}{2}\varepsilon_{ijk} F_{jk}^a$. Further, we can impose the canonical equal-time Poisson brackets:

$$\{E_\nu^a(t, \mathbf{x}), A_\mu^b(t, \mathbf{y})\} = g_{\nu\mu} \delta^{ab} \delta^{(3)}(\mathbf{x} - \mathbf{y}) \tag{9a}$$

$$\{q^a(t,\mathbf{x}), Q^b(t,\mathbf{y})\} = \delta^{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{9b}$$

$$\{\pi^{\bar a}(t,\mathbf{x}), c^{\bar b}(t,\mathbf{y})\} = \delta^{\bar a\bar b}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{9c}$$

$$\{\bar\pi^3(t,\mathbf{x}), \bar\eta(t,\mathbf{y}) = \{\pi^3(t,\mathbf{x}), \eta(t,\mathbf{y})\} = \delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{9d}$$

and all other brackets vanish identically. This allows us to look for the secondary constraints. For, if the primary constraints *just come* from our Lagrangian as relations on the canonical conjugate momenta, then the secondary ones *are to be imposed* as the stationarity conditions (Dirac, 1950, 1959, 1964):

$$\begin{aligned}\varphi_1^a(x) &:= \frac{d}{dt}E_0^a(x)\\ &= \{E_0^\alpha(x), H\}\\ &= -D_i^{ab}E_i^b + \delta^{a1}(Q^2\rho + g\pi^2\eta) - \delta^{a2}(Q^1\rho + g\pi^1\eta)\\ &\quad + \delta^{a3}(g\varepsilon^{\bar a\bar b}\pi^{\bar a}c^{\bar b} - \rho) = 0\end{aligned} \tag{10a}$$

$$\begin{aligned}\chi_1^a(x) &:= \frac{d}{dt}q^a(x)\\ &= \{q^a(x), H\}\\ &= D_\mu^{ab}j^{b\mu}\\ &= \delta^{a1}A_0^1\rho - \delta^{a2}A_0^2\rho + \delta^{a3}\dot\rho - D_i^{ab}j_i^b = 0\end{aligned} \tag{10b}$$

$$\begin{aligned}\psi(x) &:= \frac{d}{dt}\pi^3(x)\\ &= \{\pi^3(x), H\}\\ &= \pm g\varepsilon^{\bar a\bar b}\pi^{\bar a}A_0^{\bar b} \pm \frac{\pi^{\bar a}}{\rho}\varepsilon^{\bar a\bar b}m^{\bar b3} - \eta m^{33} = 0\end{aligned} \tag{10c}$$

$$\begin{aligned}\bar\psi(x) &:= \frac{d}{dt}\bar\pi^3(x)\\ &= \{\bar\pi^3(x), H\}\\ &= m^{3\bar a}c^{\bar a} + m^{33}\eta = 0\end{aligned} \tag{10d}$$

$$\begin{aligned}\varphi_2(x) &:= \frac{d}{dt}\varphi_1^a(x)\\ &= \{\varphi_1^a(x), H\} + \dot\varphi_1(x)\\ &= -\dot\rho + D_i^{3a}j_i^a + m^{3\bar a}Q^{\bar a} + m^{33}Q^3 - \varepsilon^{\bar a\bar b}A_0^{\bar b}D_i^{\bar a c}E_i^c = 0\end{aligned} \tag{10e}$$

$$\begin{aligned}\chi_2(x) &:= \frac{d}{dt}\chi_1^3(x)\\ &= \{x_1^3(x), H\} + \dot\chi_1^3(x)\\ &= m^{3a}A_0^a + \ddot\rho - D_i^{ab}j_i^b - \varepsilon^{\bar a\bar b}E_i^{\bar a}j_i^{\bar b} = 0\end{aligned} \tag{10f}$$

where $H=\int d^3x\,\mathscr{H}(x)$. One can check that there are no more secondary constraints and that they are all of second class if only the condition

$$\rho(x)m^{33}(x)\neq 0 \tag{11}$$

is satisfied at every point x.

Now we could write down the path-integral formula for the S-matrix (Senjanovic, 1976; Fradkin and Fradkina, 1978), but it would depend drastically on what statistics we have imposed on the subsidiary fields c^α, η, $\bar\eta$. Instead we propose to solve some constraints and reduce the set of independent variables. First we find the values of A_0^i, A_0^2, η (a_0^1, a_0^2, η_0), then the values of B^a (b^a), and finally the value of A_0^3 (a_0^3). Now there are only two constraints left, $\varphi_1^3=0$ and $\chi_1^3=0$, and in order to arrive at *the completely unconstrained dynamics* one should solve them as well. However, we will not pursue this; instead, we will stop here and consider *the almost unconstrained prescription.* Now our Hamiltonian density assumes the following form:

$$\begin{aligned}\mathscr{H}_0(a_0^{\bar a},\eta_0)=&\tfrac{1}{2}(E_i^a)^2+\tfrac{1}{2}(B_i^a)^2+A_i^a j_i^a+a_0^{\bar a}D_i^{\bar a b}E_i^b\\&\pm(\pi^{\bar a}/\rho)\varepsilon^{\bar a\bar b}m^{\bar b\bar c}c^{\bar c}+\bar\eta_0 m^{3\bar a}c^{\bar a}\end{aligned} \tag{12}$$

and the residual constraints are still of second class,

$$\{\chi_1^3(t,\mathbf{x}),\varphi_1^3(t,\mathbf{y})\}=m^{33}(t,\mathbf{x})\delta(\mathbf{x}-\mathbf{y})\neq 0 \tag{13}$$

This allows us to write down the phase path-integral formula for the generating functional (Senjanovic, 1976; Fradkin and Fradkina, 1978):

$$\begin{aligned}Z[j_\mu^a]:=&\int DE_i^a\,DA_i^a\,D\pi^{\bar a}\,Dc^{\bar a}\,\delta(\varphi_1^3)\,\delta(\chi_1^3)\,\text{Det}\,m^{33}\\&\times\exp i\int d^4x\,[\dot A^a E_i^a+\dot c^{\bar a}\pi^{\bar a}=\mathscr{H}_0(a_0^{\bar a},\eta_0^{\bar a})]\end{aligned} \tag{14}$$

We see that our system possesses ten independent dynamical degrees of freedom and we disagree with the arguments of Cabo and Shabad (1986) that there are only six physical modes. We connect the extra physical modes with a new phenomenon: some subsidiary scalar fields are dynamical now. Unfortunately, this posibility was overlooked in their paper.

However, we would like to rewrite the above path-integral in an explicitly convariant form. Our first trick is simple; we introduce δ-functionals in order to impose the definite values a_0^a on A_0^a fields:

$$\begin{aligned}Z[j_\mu^a]=&\int DE_i^a\,DA_\mu^a\,D\pi^{\bar a}\,Dc^{\bar a}\,\delta(A_0^a-a_0^a)\delta(\chi_1^3)\delta(\varphi_1^3)\\&\times\text{Det}\,m^{33}\exp i\int d^4x\,[\dot A_i^a E_i^a+\dot c^{\bar a}\pi^{\bar a}+\mathscr{H}_0(A_0^{\bar a},\bar\eta_0)]\end{aligned} \tag{14'}$$

Now there is a point where we have to decide whether subsidiary scalar fields $c^{\bar{a}}$ are to be treated as bosons or fermions. First we suppose that they are anticommuting variables.

Path integrals over anticommuting Grassmann variables are already well known (Itzykson and Zuber, 1980) and for example we have

$$\int D\bar{c}^3\, Dc^3 \exp i \int d^4x\, [-\bar{c}^3 m^{33} c^3 + \bar{c}^3 m^{3\bar{a}} c^{\bar{a}} - \bar{\eta}_0 m^{33} c^3]$$

$$= \mathrm{Det}\, m^{33} \exp - i \int d^4x\, \bar{\eta}_0 m^{33} c^3 \tag{15a}$$

$$\int DA_0^{\bar{a}}\, Dc^a\, \delta[(\rho A_0^{\bar{a}} - \rho a_0^{\bar{a}})\varepsilon^{\bar{a}\bar{b}}] \ldots = \int A_0^{\bar{a}}\, D\pi_\delta^{\bar{a}}(A_0^{\bar{a}} - a_0^{\bar{a}}) \cdots \tag{15b}$$

where

$$\bar{c}^{\bar{a}} = -\varepsilon^{\bar{a}\bar{b}}\pi/\rho$$

Our next step is again obvious; we introduce fields A_0^3, Q^a as the dummy variables:

$$\prod_x \delta(\chi_1^3)\, \delta(-\rho A_0^2 + \rho a_0^2)\, \delta(\rho A_0^1 - \rho a_0^1)\, \delta(\varphi_1^3)$$

$$= \int DA_0^3\, DQ^a \exp i \int d^4x\, [Q^a(x)\chi_1^a(x) - A_0^3(x)\varphi_1^3(x)] \tag{15c}$$

All this leads to the following form of our path-integral formula for the generating functional:

$$Z[j_\mu] = \int DE_i^a\, DA_\mu^a\, DQ^a\, D\bar{c}^a\, Dc^a \exp i \int d^4x\, [E_i^a \dot{A}_i^a + \rho\varepsilon^{\bar{a}\bar{b}} \bar{c}^{\bar{a}} \dot{c}^{\bar{b}} - \mathcal{H}_1] \tag{16}$$

where we have

$$\begin{aligned} \mathcal{H}_1 = {} & \tfrac{1}{2}(E_i^a)^2 + \tfrac{1}{2}(B_i^a)^2 + A_i^a j_i^a - A_0^a(D_i^{ab} E_i^b + \rho\delta^{a3}) \\ & - \rho g(\bar{c}^{\bar{a}} A_0^{\bar{a}} c^9 - \bar{c}^a c^a A_0^9) - \bar{c}^a m^{ab} c^b \\ & + \rho\varepsilon^{\bar{a}\bar{b}} Q^{\bar{a}} A_0^{\bar{b}} + Q^a(D_i^{ab} \dot{j}_i^b - \dot{\rho}\delta^{a3}) \end{aligned}$$

Now we can perform the Gaussian integral over E_i^a and arrive at the Feynman path integral:

$$Z[j_\mu] = \int DA_\mu^a\, DQ^a\, D\bar{c}^a\, Dc^a \exp i \int d^4x\, \bar{L}_{\mathrm{mod}}(x) \tag{17}$$

where the modified Lagrangian density $\bar{L}_{\rm mod}$ is given by equation (6). However, the above formula is not quite satisfactory, since the external current j_μ is not arbitrary. In order to arrive at the formula for an arbitrary current J_μ we change the integration variables:

$$A_\mu^a \to R_0^{ab} A_\mu^b + \frac{1}{g}\, \varepsilon^{abc} R_0^{bd} \partial_\mu R_0^{cd} \tag{18a}$$

$$c^a \to R_0^{ab} c^b \tag{18b}$$

$$\bar{c}^a \to R_0^{ab} \bar{c}^b \tag{18c}$$

$$Q^a \to R_0^{ab} Q^b \tag{18d}$$

where the orthogonal matrix R_0^{ab} is connected with the unitary matrix U_0 [from equation (4)] by the relation

$$R_0^{ab} := \mathrm{Tr}\{\lambda^a U_0 \lambda^b U_0^{-1}\} \tag{19}$$

From the above definition of R_0, using the properties of λ, one can easily find the following relations:

$$R_0^{ab} R_0^{ac} = \delta^{bc} \tag{20a}$$

$$\varepsilon^{abc} R_0^{ai} R_0^{bj} R_0^{ck} = \varepsilon^{ijk} \tag{20b}$$

Thus, the Jacobian of the transformation (18) is equal to unity. This leads to the following relation for the generating functional:

$$Z[j_\mu] = \exp i\Phi(R_0)\, Z[J_\mu] \tag{21}$$

where

$$\Phi(R_0) = \int d^4x\, (\varepsilon^{abc} J^{a\mu} R_0^{bd} \partial_\mu R_0^{cd}) \tag{22}$$

$$Z[J_\mu] = \int DA_\mu^a\, DQ^a D\bar{c}^a\, Dc^a \exp i \int d^4x\, L_{\rm mod} \tag{23}$$

here $L_{\rm mod}$ is the modified Lagrangian density given by equation (1). Thus, we have killed two birds with one stone—we have found both the path-integral formula for the generating functional in the presence of arbitrary external sources and its transformation law.

Now, we return to our earlier problem of the scalar field statistics and suppose that they are bosons. The analogs of equations (15a) and (15b) will contain the inverse power of Det m^{33} and the factor Det ρ^{-4} on their right-hand sides, respectively. The rest of the analysis will follow similar lines as before; our final formula is

$$Z[j_\mu] = \int DA_\mu^\alpha \, DQ^a \, D\bar{c}^a \, Dc^a \, \mathrm{Det}\left[\frac{m^{33}}{\rho^2}\right]^2 \exp i \int d^4x \, \bar{L}_{\mathrm{mod}}(x) \quad (24)$$

or, in the generic case,

$$Z[J_\mu] = \int DA^a \, DQ^a \, D\bar{c}^a \, Dc^a \, \mathrm{Det}\left[\frac{J_0^a J_0^b m^{ab}}{(J_0^c)^2 (J_0^d)^2}\right]$$
$$\times \exp i \int d^4x \, L_{\mathrm{mod}}(x) \quad (24')$$

Note the very surprising phenomenon that a wrong statistics of the subsidiary fields breaks the Lorentz invariance of our final formula. Our starting point was a Lorentz-invariant quantity (no matter what statistics the subsidiary fields obeyed) if only external sources J_μ transformed as Lorentz vectors. However, the canonical quantization procedure favors a time coordinate; thus, an explicit Lorentz invariance comes out in such a subtle way.

Unfortunately, we cannot give any intuitive explanation of our results, we just stress their uniqueness. For, if one quantizes gauge field theory in covariant gauges (Kugo and Oijima, 1979), then *all ghost fields are dynamical* and Lorenta invariance of the S-matrix formula will not depend on the ghost field statistics. Also, the source term modification when *all ghosts are nondynamical*, e.g., in the Abelian scalar electrodynamics (see Appendix A) or in the $SU(2)$ Yang-Mills fields coupled to external current ($J_0^\alpha = 0$, $J_i^a \neq 0$; see Appendix B) will not give rise to such a pnenomenon. Thus, we can argue that *only the simultaneous presence of dynamical and nondynamical ghost degrees of freedom will lead to a final result which is sensitive to the statistics of ghosts.*

In closing this section, we point out the BRS-like supersymmetry (Becchi *et al.*, 1976) of our path-integral formula (23):

$$\delta_{\mathrm{BRS}} A_\mu^\alpha = D_\mu^{ab} c^b \lambda \quad (25a)$$

$$\delta_{\mathrm{BRS}} \bar{c}^a = -Q^a \lambda \quad (25b)$$

$$\delta_{\mathrm{BRS}} c^\alpha = -\tfrac{1}{2} \varepsilon^{abc} c^b c^c \lambda \quad (25c)$$

$$\delta_{\mathrm{BRS}} Q^a = c^a \lambda \quad (25d)$$

where λ is an infinitesimal, constant parameter. There is a difference between the actual invariance and the usual one. Here we also observe the supersymmetry invariance in the presence of external sources, and usually source terms break the BRS invariance. A detailed discussion of this problem is postponed for our next paper.

3. CANONICAL QUANTIZATION OF THE $SU(2)$ YANG–MILLS FIELD THEORY WITH NAIVE SOURCE TERMS

In this section we sketch some features of the canonical path-integral procedure in the case of the usual (naive) source terms (Schwinger, 1951). Now our Lagrangian density is simple:

$$L_{\text{naive}} = -\tfrac{1}{4}F^{a\mu\nu}F^{a}_{\mu\nu} + A^{\alpha\mu}J^{\alpha}_{\mu} \tag{26}$$

We will carry out our analysis in the case when the external charges take the following form:

$$J^{a}_{0} = \rho\delta^{a3} \tag{5'}$$

and then transform our final formula back to the generic case. First we look for the canonical conjugate momenta:

$$E^{a}_{i}(x) := \delta S_{\text{naive}}/\delta \dot{A}^{a}_{i}(x) \tag{27a}$$

$$E^{a}_{0}(x) := \delta S_{\text{naive}}/\delta \dot{A}^{a}_{0}(x) \tag{27b}$$

where

$$S_{\text{naive}} = \int d^{4}x\, L_{\text{naive}}(x)$$

and we write the resulting Hamiltonian density:

$$\begin{aligned}\mathscr{H} &= E^{a}_{i}\dot{A}^{a}_{i} - L_{\text{naive}} + v^{a}E^{a}_{0}\\ &= \tfrac{1}{2}(E^{a}_{i})^{2} + \tfrac{1}{2}(B^{a}_{i})^{2} + A^{a}_{i}j^{a}_{i} - A^{a}_{i}(D^{ab}_{i}E^{b}_{i} + \rho\delta^{a3}) + v^{a}E^{a}_{0}\end{aligned} \tag{28}$$

Further, we can impose the canonical Poisson brackets:

$$\{E^{a}_{\mu}(t,\mathbf{x})A^{b}_{\nu}(t,\mathbf{y})\} = g_{\mu\nu}\delta^{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{29}$$

(all other brackets are equal to zero) and find the set of secondary constraints:

$$\varphi^{a}_{1} = \{E^{a}_{0}, H\} = -D^{ab}_{i}E^{b}_{i} - \rho\delta^{a3} = 0 \tag{30a}$$

$$\varphi^{a}_{2} = \{\varphi^{a}_{2}, H\} + \dot{\varphi}^{a}_{2} = -g\varepsilon^{abc}A^{c}_{0}D^{bd}_{i}E^{d}_{i} - \delta^{a3} + D^{ab}_{i}j^{b}_{i} = 0 \tag{30b}$$

$$\begin{aligned}\varphi_{3} = \{\varphi^{3}_{2}, H\} + \dot{\varphi}^{3}_{2} &= g\varepsilon^{3bc}\varepsilon^{bde}A^{d}_{0}A^{c}_{0}D^{ea}_{i}E^{a}_{i}\\ &\quad + \ddot{\rho} + D^{9a}_{i}j^{a}_{i} + \varepsilon^{9cb}j^{b}_{i}(-E^{c}_{i} + D^{cd}_{i}A^{d}_{0}) = 0\end{aligned} \tag{30c}$$

One can easily convince oneself that all constraints are of the second class, because we generally have

$$\text{Det}\{\Theta^{i}, \Theta^{j}\}_{\Theta=0} = \text{Det}[g^{3}\rho^{2}(x)m^{33}(x)]^{2} \neq 0 \tag{31}$$

where

$$\Theta^{i} = \{E^{a}_{0}, \varphi^{a}_{1}, \varphi^{a}_{2}, \varphi_{3}\}, \qquad i = 1, 2, \ldots, 10$$

$$m^{33}(x) = \varepsilon^{3cd}j^{c}_{i}(x)D^{d3}_{i}(x)$$

Now we can write down the phase path-integral formula for our system (Senjanovic, 1976; Fradkin and Fradkina, 1978):

$$Z[j_\mu]=\int DE^a_\mu\, DA^a_\mu\, \delta(\Theta^i)[\mathrm{Det}\{\Theta^i,\Theta^j\}_{\Theta=0}]^{1/2}$$

$$\times \exp i \int d^4x\, [E^a_i \dot{A}^a_i - \mathscr{H}] \tag{32}$$

and after some simple calculational tricks we arrive at the Feynman path integral:

$$Z[j_\mu]=\int DA^a_\mu\, \delta(D^{3b}_i j^b_i)\, \mathrm{Det}[g^3 \rho m^{33}]$$

$$\times \exp i \int d^4x\, (L + A^3_0 \rho - A^a_i j^a_i) \tag{32'}$$

Finally we change the integral variables:

$$A^a_\mu = R^{ab}_0 A^b_\mu + \frac{1}{g}\,\varepsilon^{abc} R^{bd}_0 \partial_\mu R^{cd}_0 \tag{33}$$

where the matrix R_0 is given by equation (19), and we arrive at the final formulas

$$Z[j_\mu]=Z[J_\mu]\exp i\Phi(R_0) \tag{34}$$

$$Z[J_\mu]=\int DA^a_\mu\, \delta(J^a_0 D^{ab}_\mu J^{b\mu})\, \mathrm{Det}[g^3 J^a_0 J^b_0 m^{ab}]$$

$$\times \exp i \int d^4x\, (L + A^a_\mu J^{a\mu}) \tag{35}$$

Here again, we have obtained a final S-matrix formula which explicitly violates Lorentz invariance through *our initial Lagrangian* (25) *was Lorentz-invariant.* One can also notice that expression (35) is singular when $J^a_0 \to 0$ and one cannot smoothly pass to the desired result (Appendix B, Cabo and Shabad, 1986):

$$Z[J_i]=\int DA^a_\mu\, \delta(D^{ab}_i J^b_i)\, \mathrm{Det}\, m^{ab} \exp i \int d^4x (L - A^a_i J^a_i) \tag{36}$$

We argue that the lack of Lorentz invariance and the above discrepancy are evidence for the necessity of source term modification.

APPENDIX A

Here we analyze a simple case of an Abelian gauge field theory in the presence of arbitrary source terms—scalar electrodynamics. We propose to invert the previous order of presentation and discuss the case of the usual source terms first. Thus, we start with the following Lagrangian density:

$$L=-\tfrac{1}{4}f_{\mu\nu}f^{\mu\nu}+|D_\mu\phi|^2+F(|\phi|^2)+A^\mu J_\mu+\eta^*\phi+\eta\phi^* \tag{A1}$$

where $f_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and $D_\mu=\partial_\mu+ieA_\mu$.

First we look for the canonical conjugate momenta:

$$E_i:=\delta S/\delta\dot{A}_i=f_{0i} \tag{A2a}$$

$$E_0:=\delta S/\delta\dot{A}_0\equiv 0 \tag{A2b}$$

$$\pi\pi\,\delta S/\delta\dot{\phi}=(D_0\phi) \tag{A2c}$$

$$\varepsilon:=\delta S/\delta\dot{\phi}^*=D_0\phi \tag{A2d}$$

and write down the Hamiltonian density:

$$\begin{aligned}\mathscr{H}&=E_i\dot{A}_i+\pi^*\dot{\phi}+\pi\dot{\phi}^*-L-vE_0\\&=\tfrac{1}{2}(E_i)^2+\tfrac{1}{2}(B_i)^2+\pi^*\pi+|D_i\phi|^2-A_0[\partial_iE_i-ie(\pi^*\phi-\pi\phi^*)-J_0]\\&\quad+A_iJ_i-\phi^*\eta-\phi\eta^*-F(|\phi|^2)+vE_0\end{aligned} \tag{A3}$$

If now we impose the canonical Poisson brackets

$$\{E_\mu(t,\mathbf{x}),\,A_\nu(t,\mathbf{y})=g_{\mu\nu}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{A4a}$$

$$\{\pi^*(t,\mathbf{x}),\,\phi^*(t,\mathbf{y})\}=\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{A4b}$$

$$\{\pi(t,\mathbf{x}),\,\phi(t,\mathbf{y})\}=\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{A4c}$$

(all other brackets are equal to zero), then we will obtain the following sequence of secondary constraints:

$$\varphi_1=\{E_0,H\}=\partial_iE_i-ie(\pi^*\phi-\pi\phi^*)-J_0=0 \tag{A5a}$$

$$\varphi_2=\{\varphi_1,H\}+\dot{\varphi}_1=-\partial_iJ_i+ie(\phi^*\eta-\phi\eta^*)=0 \tag{A5b}$$

$$\begin{aligned}\varphi_3&=\{\varphi_2,H\}+\dot{\varphi}_2\\&=-\partial_i\dot{J}_i+ie(\phi^*\eta-\phi\eta^*)+ie(\pi\eta^*-\pi^*\eta)\\&\quad-e^2A_0(\phi^*\eta+\phi\eta^*)=0\end{aligned} \tag{A5c}$$

Further, all constraints are of second class, because for arbitrary sources η, η^* we generally have

$$\mathrm{Det}\{\Theta^i,\Theta^j\}_{\Theta=0}=\mathrm{Det}[e^8(\phi\eta^*+\phi^*\eta)^4]\neq 0 \tag{A6}$$

where $\Theta^i=\{E_0,\varphi_1,\varphi_2,\varphi_3\}$.

Now we are in a position to write down the phase path-integral formula for the generating functional (Senjanovic, 1976; Fradkin and Fradkina,

1978) and to perform some simple calculations:

$$
\begin{aligned}
&Z[J_\mu, \eta, \eta^*] \\
&\quad = \int DA_\mu\, DE_\mu\, D\pi\, D\pi^*\, D\phi\, D\phi^*\, \delta(\Theta^i)[\mathrm{Det}\{\Theta^i, \Theta^j\}]^{1/2} \\
&\qquad \times \exp i \int d^4x\, (E_\mu \dot{A}^\mu + \pi\dot{\phi}^* + \pi^*\dot{\phi} - \mathcal{H}) \\
&\quad = \int DA_i\, DE_i\, D\pi\, D\pi^*\, D\phi\, D\phi^* \delta(\varphi_1)\delta(\varphi_2)\, \mathrm{Det}[e^2(\eta^*\phi + \eta\phi^*)] \\
&\qquad \times \exp i \int d^4x\, (E_i \dot{A}_i + \pi\dot{\phi}^* + \pi^*\dot{\phi} - \mathcal{H}(A=0) \\
&\quad = \int DA_\mu\, D\phi\, D\phi^* \delta(\varphi_2)\, \mathrm{Det}[e^2(\eta\phi^* + \eta^*(\phi)]\, \exp i \int d^4x\, L \qquad \text{(A7)}
\end{aligned}
$$

One can notice the close analogy between the final expression and a result of the usual Faddeev–Popov prescription (Faddeev and Popov, 1967); we have the gauge condition

$$\varphi_2 = \partial_\mu J^\mu - ie(\eta^*\phi - \eta\phi^*) = 0 \qquad \text{(A8)}$$

and the ghost field contribution

$$\mathrm{Det}[e^2(\eta^*\phi + \eta\phi^*)] \qquad \text{(A9)}$$

However, we would like to stress that this similarity is superficial, because our system is not singular (gauge-invariant), though its dynamics is still constrained. One should also notice that while the Faddeev–Popov prescription holds for a system with first-class constraints, our model possesses the second-class ones.

We can also introduce the modified source terms:

$$
\begin{aligned}
L_{\mathrm{mod}} = &-\tfrac{1}{2} f^{\mu\nu} f_{\mu\nu} + |D_\mu\phi|^2 - F(|\phi|^2) + A^\mu J_\mu + \phi\eta^* + \phi^*\eta \\
&+ Q[\partial_\mu J^\mu - ie(\eta\phi^* - \eta^*\phi)] + e^2\bar{c}(\eta\phi^* + \eta^*\phi)c \qquad \text{(A10)}
\end{aligned}
$$

but this will change substantially nothing—all extra degrees of freedom are nondynamical:

$$\delta L_{\mathrm{mod}}/\delta\dot{Q}(x) = \delta L_{\mathrm{mod}}/\delta\dot{c}(x) = \delta L_{\mathrm{mod}}/\delta\dot{\bar{c}}(x) \equiv 0 \qquad \text{(A11)}$$

One can check that the new secondary constraints just cancel the subsidiary fields Q, c, and $\bar{c}$ and the final formula will not be changed at all. It is a trivial observation that in the present case the statistics of subsidiary fields [in equation (A10)] does not influence the Lorentz invariance of the S-matrix formula.

APPENDIX B

The quantization of the $SU(2)$ Yang-Mills field theory with the usual source term $A_i^a J_i^a$ has been successfully performed by Cabo and Shabad (1986), so we will just quote their final result:

$$Z[J_i] = \int DA_\mu^a \left[(D_i^{ab} J_i^b) \operatorname{Det}(m^{ab}) \exp i \int d^4x \, (L - A_i^a J_i^a) \right. \tag{B.1}$$

Here again, as in the Abelian case, this result looks plausible but nevertheless we will find out the consequences of the source term modification.

Now our modified Lagrangian density looks like

$$L_{\text{mod}} = -\tfrac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} - A_i^a J_i^a - Q^a D_i^{ab} J_i^b + \bar{c}^a m^{ab} c^b \tag{B2}$$

and it is easy to check that all subsidiary fields are nondynamical:

$$\delta L_{\text{mod}} / \delta \dot{Q}^a(x) = \delta L_{\text{mod}} / \delta \dot{c}^a(x) = \delta L_{\text{mod}} / \delta \dot{\bar{c}}^a(x) = 0 \tag{B3}$$

Thus, we again observe that they disappear from the unconstrained dynamics either identically or because the secondary constraints

$$D_i^{ab} J_i^b = 0 \tag{B4a}$$

$$m^{ab} c^b = 0 \tag{B4b}$$

are to be imposed. That is why the source term modification does not introduce any change in the final S-matrix formula.

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